Empirical Calibration of Exoplanet Tidal Dissipation (”Q”)

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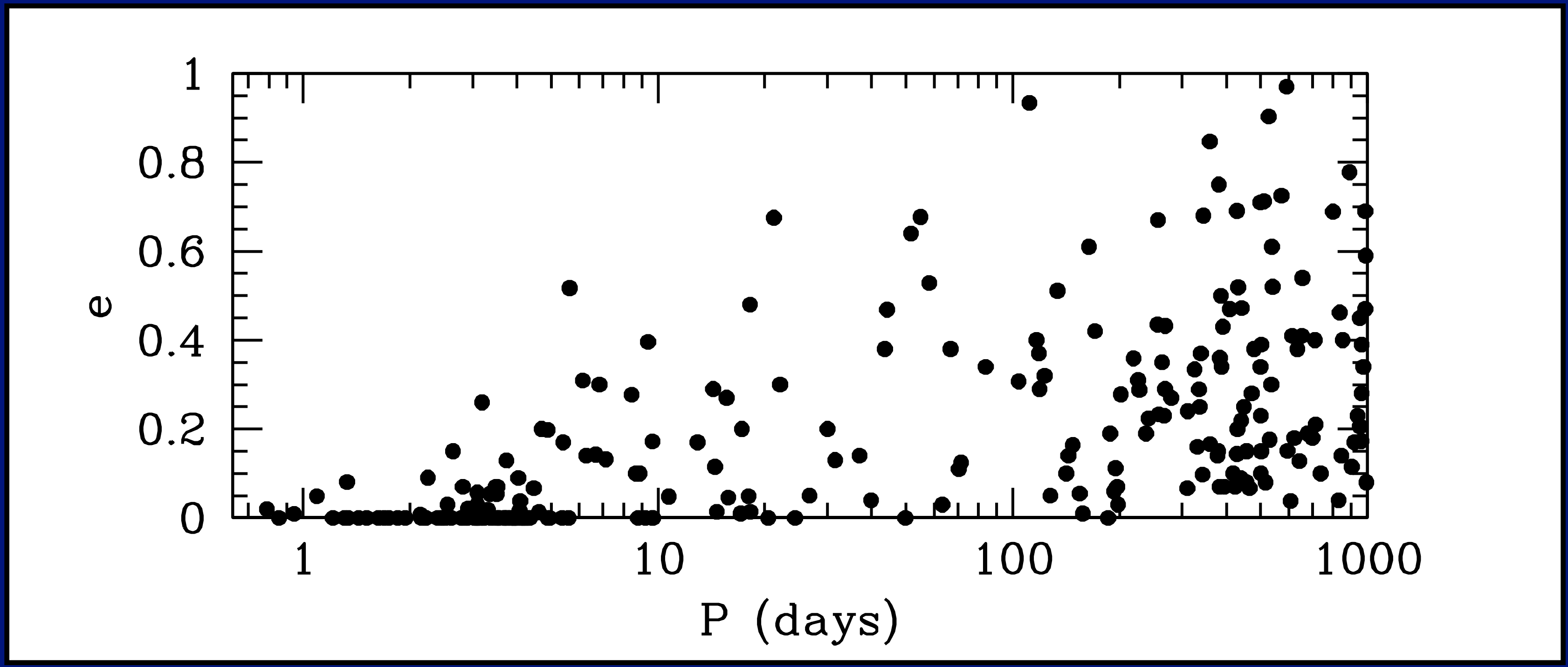
Tidal Dissipation is clearly at work in the Exoplanet systems



Tidal Circularisation (yes)
Tidal Decay of orbit due to stellar tides (maybe)
Tidal Inflation of some hot Jupiters (maybe)

Q fatigue

Discussions of tidal effects fall into two broad classes:

One class involves discussions of different physical mechanisms (dynamical tides, resonant effects, nonlinear effects etc), but there is a second class involving equilibrium tide theory, with an author’s choice of Q’.

Personally, I have found it hard to make sense of the latter class of discussion, and I have no clue which Q’ assumptions are reasonable, and which are not.

Empirical Calibration of Equilibrium Tide Theory

Eggleton, Kiseleva & Hut (1998) Ap J 499, 853-870

This formalism has several advantages.

It is appropriate for use at large eccentricities

It is derived from a more physical assumption than "constant phase lag" or "constant time lag", although it ends up being equivalent to the latter.

$$\dot{E} = -\sigma \frac{\partial q_{ij}}{\partial t} \frac{\partial q_{ij}}{\partial t} ,$$

Tidal dissipation constant - this is what I will calibrate

Dissipation in the Star and the Planet

We will describe evolution of the orbit due to tidal dissipation in both the star and the planet. We will assume the planet is tidally synchronised, but not the star.

Basic features of the evolution: Tides in planet will dominate the circularisation, and tides in the star will act to cause orbital decay even in the limit of a circular orbit (because of the transfer of angular momentum to the spin of the star)

We thus have two parameters to constrain:

σ_p – the dissipation constant in the planet

$\sigma_\star$ – the dissipation constant in the star

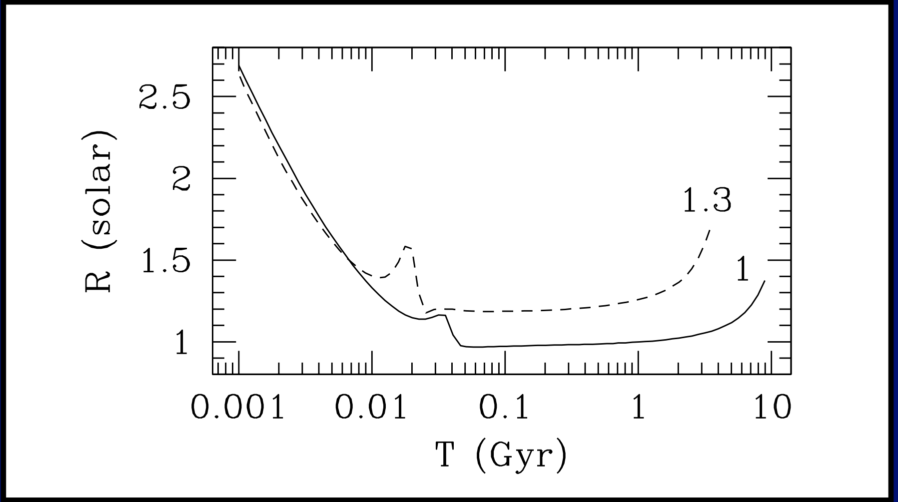
Empirical Calibration of Stellar Dissipation

Meibom & Mathieu (2005) ApJ 620, 970
provide a sample of stellar clusters in which the "circularisation period" of equal mass G-star binaries has been determined.

Calibrate our model using these binaries - assume equal mass, and that both components are circularised.

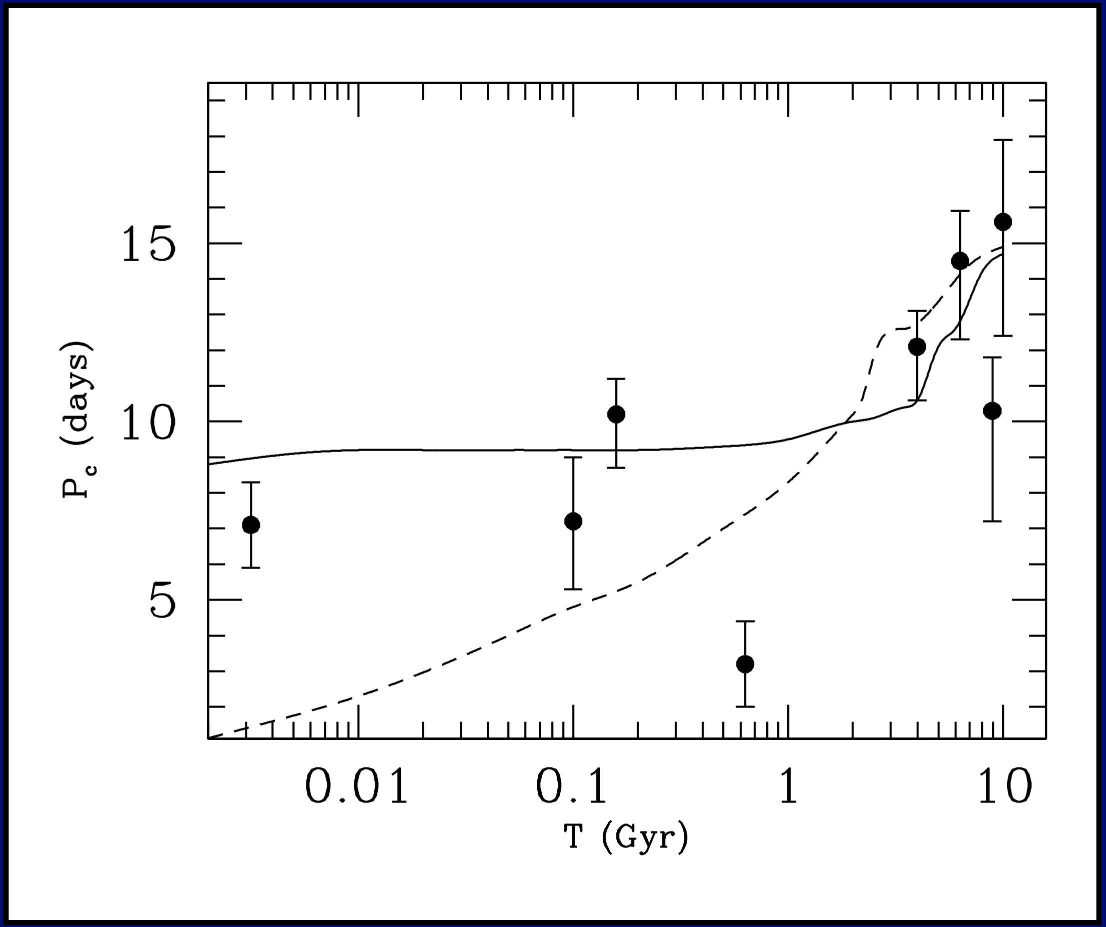
It is important to include the pre-main sequence stage, otherwise one would overestimate the amount of dissipation in a regular main sequence G star.

Baraffe et al (1998) A&A 337, 403
for evolution of stellar radius as a function of mass



Calibration of σ_* works if we include pre-MS evolution

We can perform the same exercise using a constant radius of 1 solar radius. If we calibrate it on old systems, we underpredict the rate of circularisation in young systems.

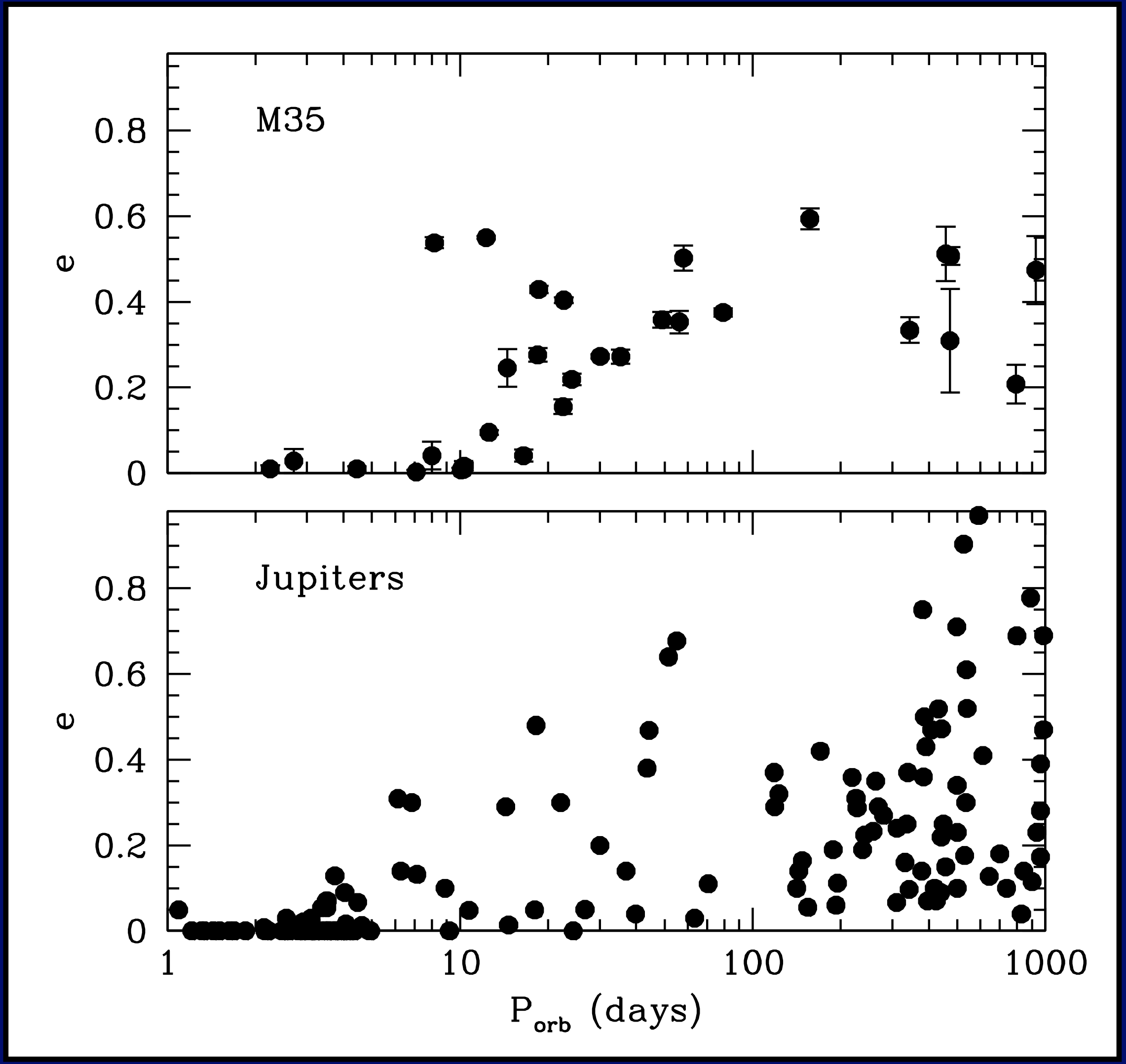


This reduces the estimated dissipation in the star by a factor of 8

Calibration of σ_P keeping σ_* constant

We restrict our sample to planets within a factor 3 of Jupiter mass to start.

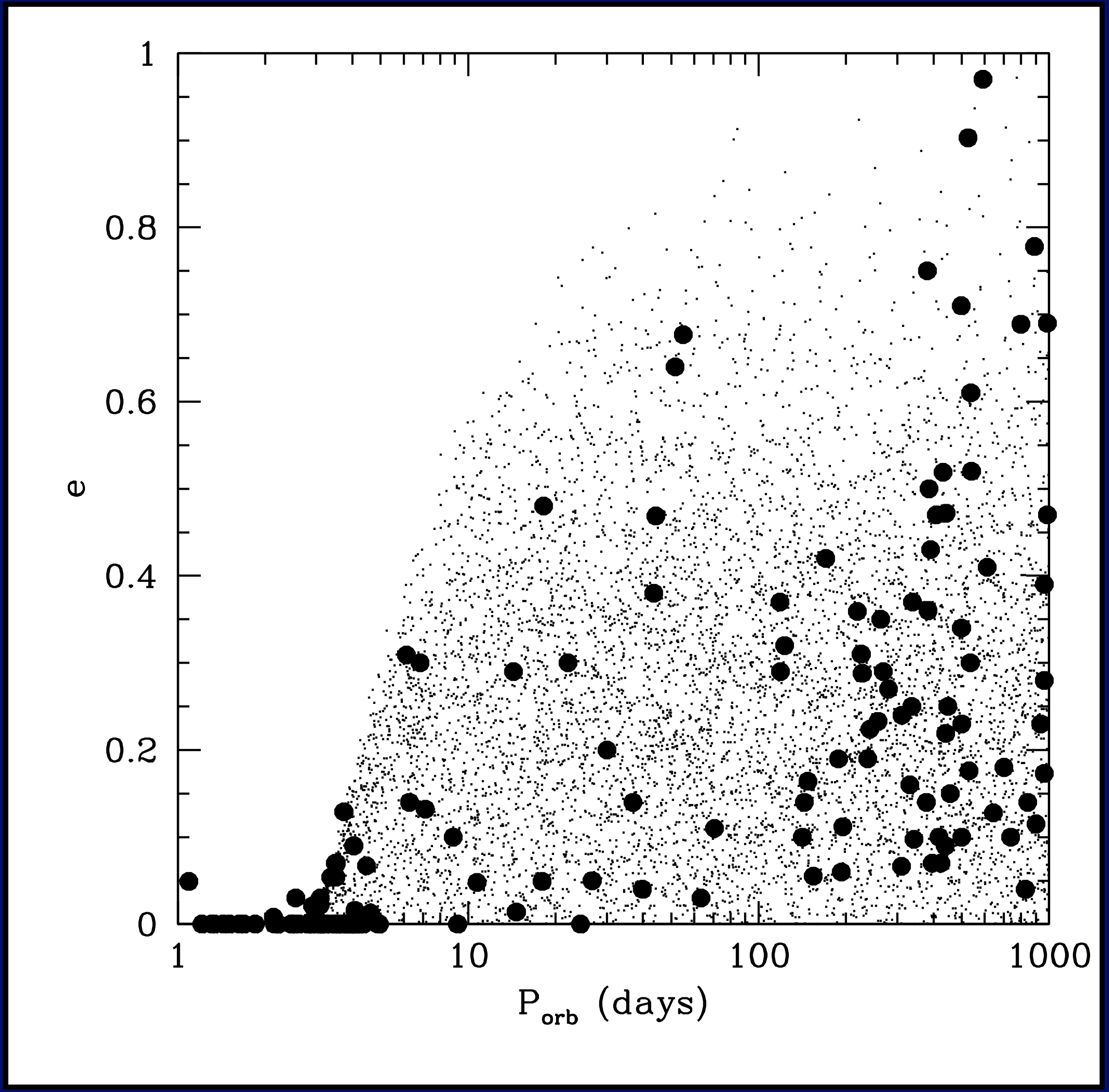
We want to repeat the same exercise as for the stellar clusters. We adopt an original e distribution based on the distribution of e at P>100 days, and a logarithmic original period distribution.



Calibration of σ_P keeping σ_* constant

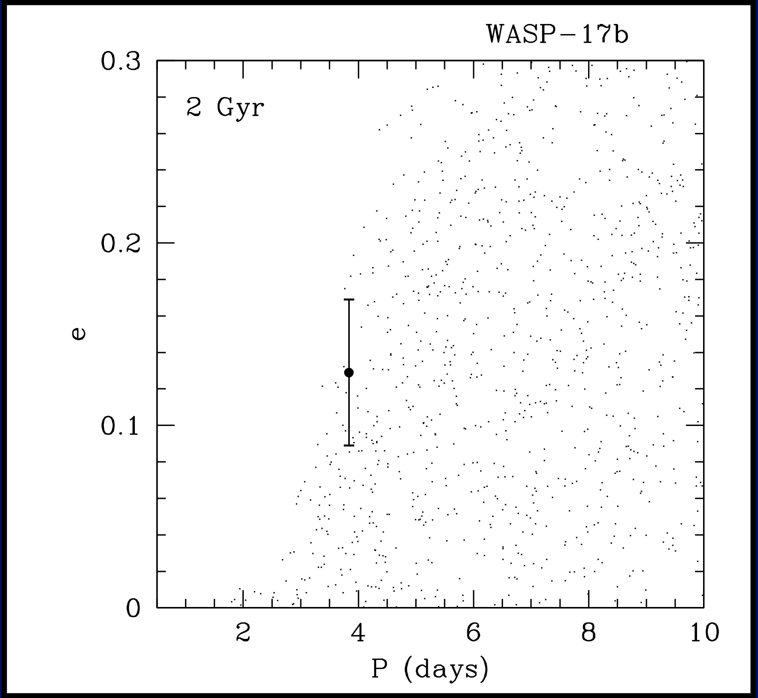
We also have to include treatments of planetary radius evolution under the effects of insolation. We use the (solar composition) models described in Hansen & Barman (2007) ApJ 671,861

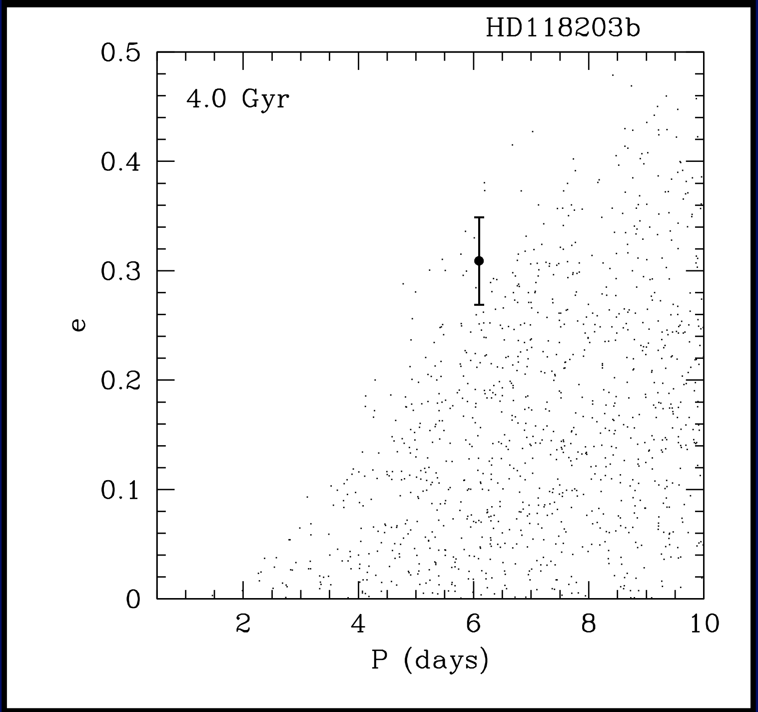
The result yields a pretty good description of the envelope of the observed eccentricity distribution.



The planet data is a relatively heterogenous sample, so it worth doing the same estimate for each individual system that defines the P-e envelope, with the correct masses and ages.

The result is that one can find a consistent solution for the systems that make up the data on the P-e plot, yielding a well-constrained value for σ_P





Dissipation expressed in terms of Q'

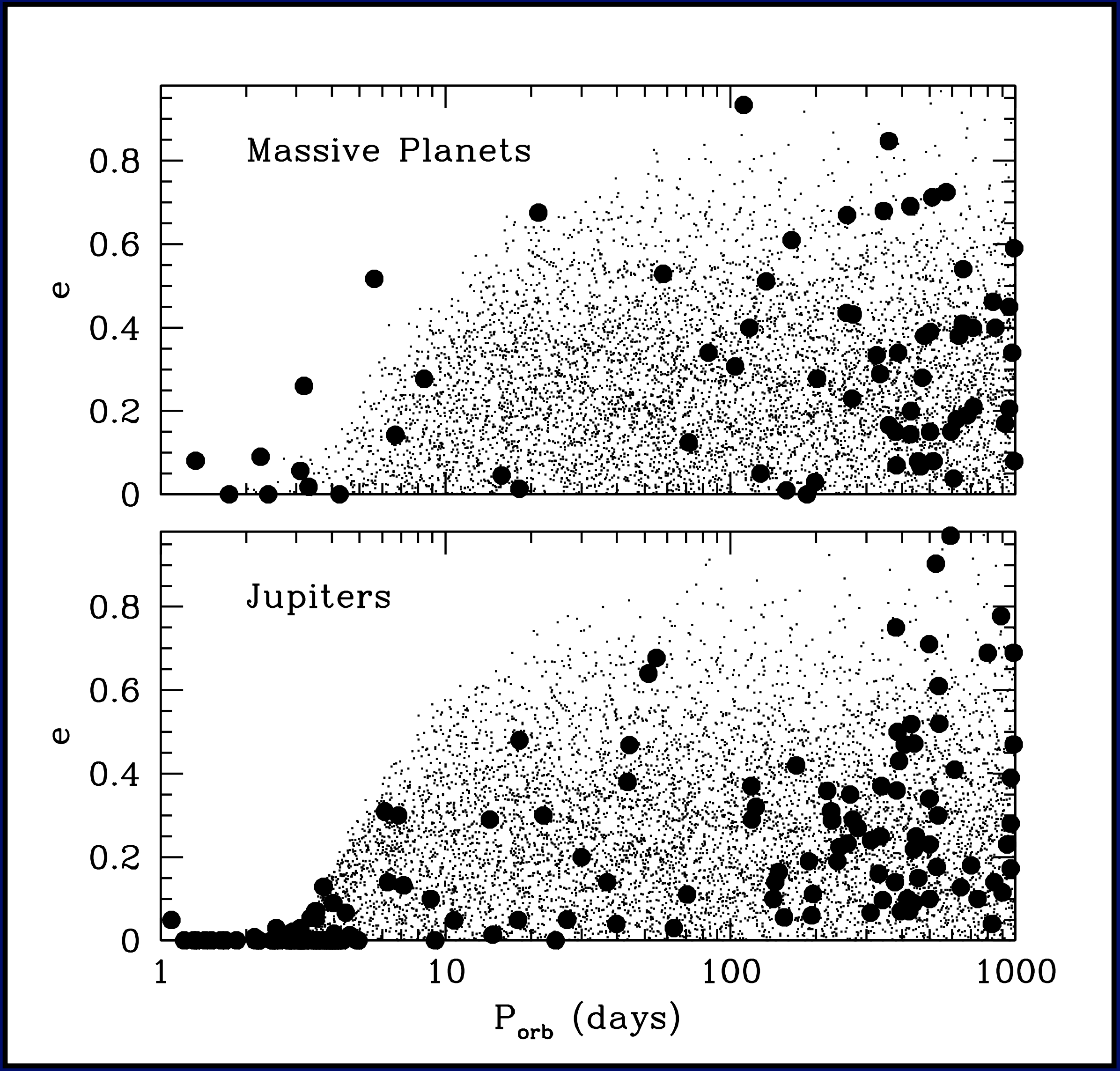
We can express the result fitted expressions in terms of the more commonly used Q' values.

$$Q'_p = 9 \times 10^6 \left(\frac{a}{0.05AU}\right)^{3/2} \left(\frac{R_p}{R_J}\right)^{-5}$$

$$Q'_* = 1.4 \times 10^6 \left(\frac{a}{0.05AU}\right)^{3/2} \left(\frac{R_*}{R_\odot}\right)^{-5}$$

The functional form of these equations can be understood by the analogy with the driven damped harmonic oscillator model, in which Q is inversely proportional to the driving frequency, as long as this is small compared to the natural frequencies of the oscillator (e.g. Greenberg 2009, ApJ 698, L42)

The model begins to break down for more massive planets, however...



Same dissipation as before, but now for an average planet mass of 7 Jupiters.

Stellar tides increase linearly with planet mass, and planet tides scale inversely with planet mass...

Stellar Tides have to be weaker!

The basic problem with the more massive planets is that the stellar tides are too strong - they swallow the planets too quickly at the observed locations

$$\frac{T_p}{T_*} = 0.5 \left(\frac{M_p/M_*}{10^{-3}} \right)^2$$

for the current calibrations of tidal dissipation in star and planet, so, for a system like XO-3b, the stellar tide is 100 times more important than for a Jupiter mass planet.

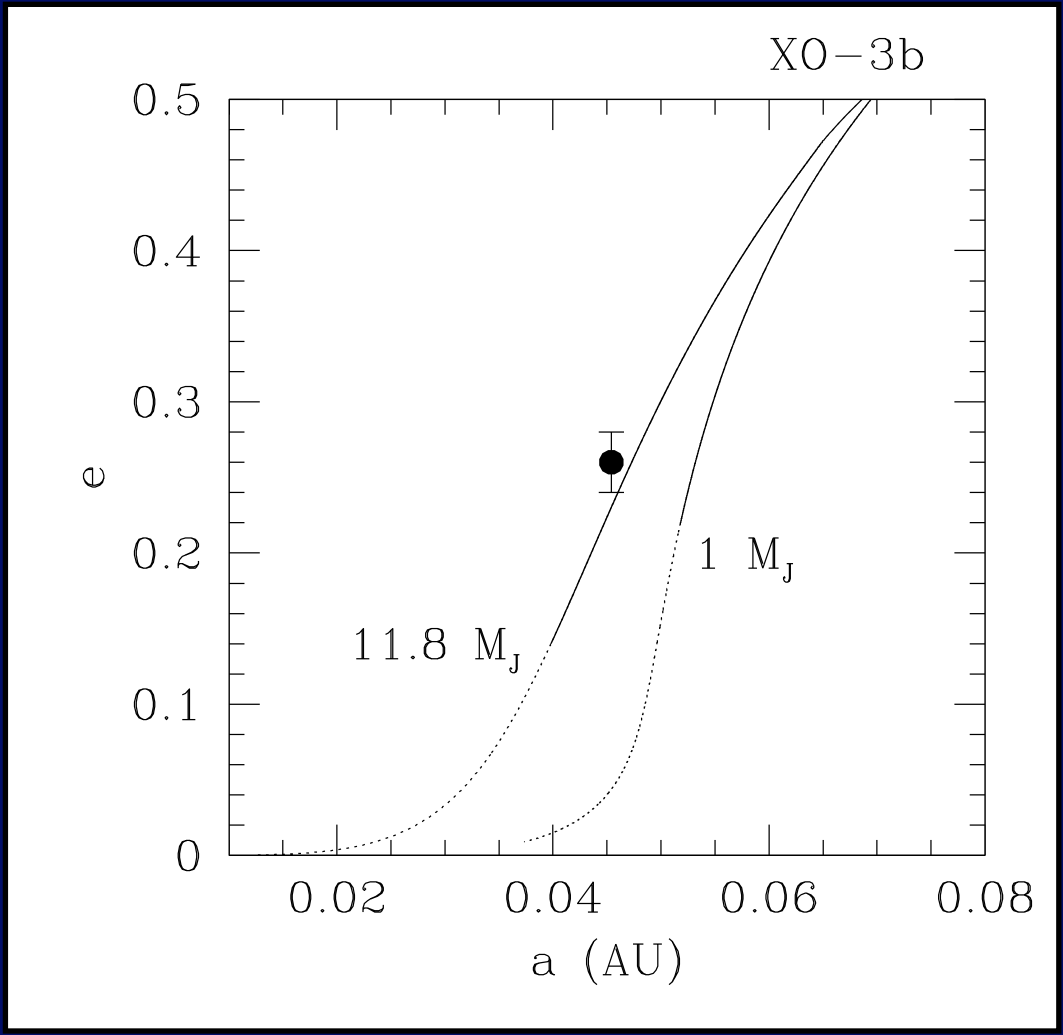
In this model, the fact that more massive planets have a higher P-e "envelope", is due to the fact that the tidal circularisation time is longer than for lower mass planets. The greater sensitivity of massive planets to stellar tides also places a stronger limit on their strength.

We can recalibrate by fitting to the systems that define the P-e envelope for more massive stars.

Recalibrating (mostly) our stellar dissipation yields revised numbers

$$Q'_p = 1.2 \times 10^7 \left(\frac{a}{0.05AU}\right)^{3/2} \left(\frac{R_p}{R_J}\right)^{-5}$$

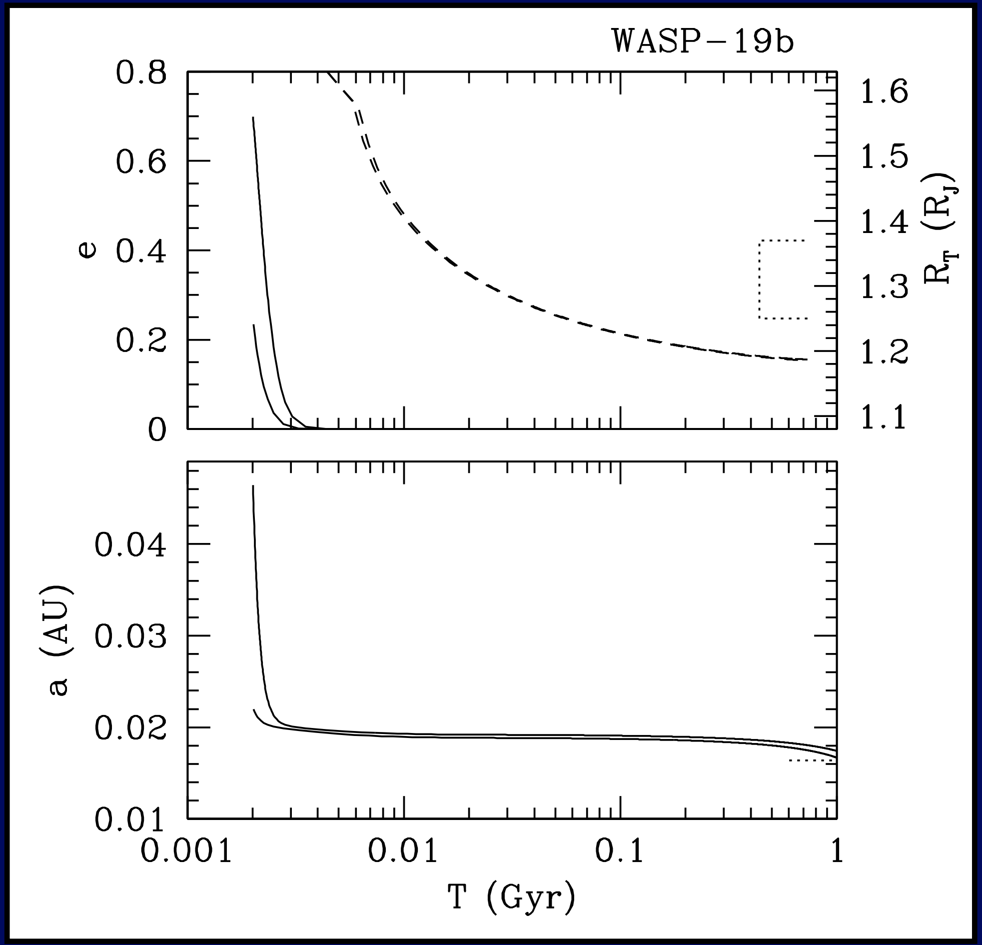
$$Q'_* = 6.4 \times 10^7 \left(\frac{a}{0.05AU}\right)^{3/2} \left(\frac{R_*}{R_\odot}\right)^{-5}$$



The more massive planets spiral in faster but damp eccentricity at a roughly equivalent rate - this shifts the envelope to slightly lower P

The Closest Exoplanets

WASP-19b is eminently stable with this level of stellar dissipation. We can find progenitor systems with a range of semi-major axes and eccentricity. The eccentricity is damped within a few million years, but the stellar dissipation is slow enough that it takes about a Gyr to swallow the planet.

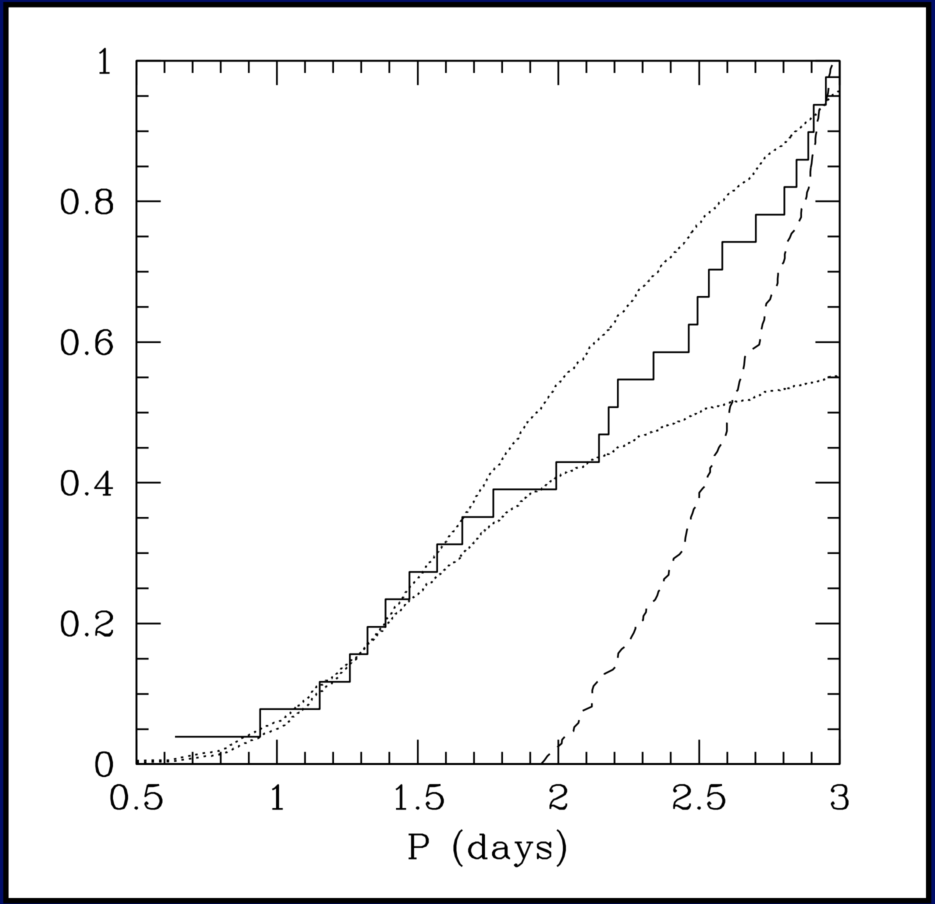


We do have some trouble matching the radius though, since the eccentricity is damped so rapidly, tidal heating is not important, and irradiation can get us only about 1.2 Rj, even with the transit radius effect included.

Q_* and the tail of Scorching Hot Jupiters

The rate at which planets are spiralling into the star will presumably also determine how steep the distribution of stars is with semi-major axis at small separations.

We can compare the cumulative distribution of Jupiter mass planets as a function of period (no super-earths or brown dwarfs) against the model. The model with reduced tidal dissipation fits well, while the older value yields too steep a rise.

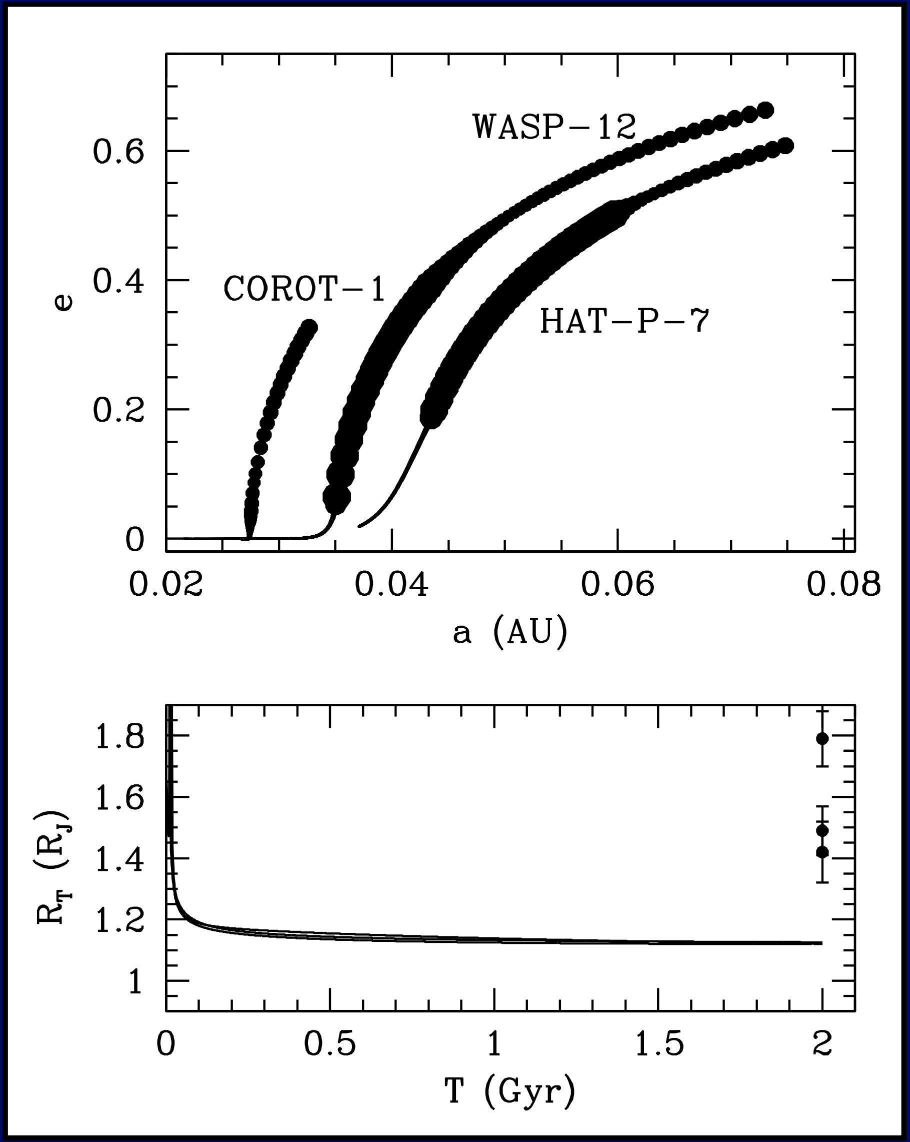


Tidal Inflation of Radii seems problematic

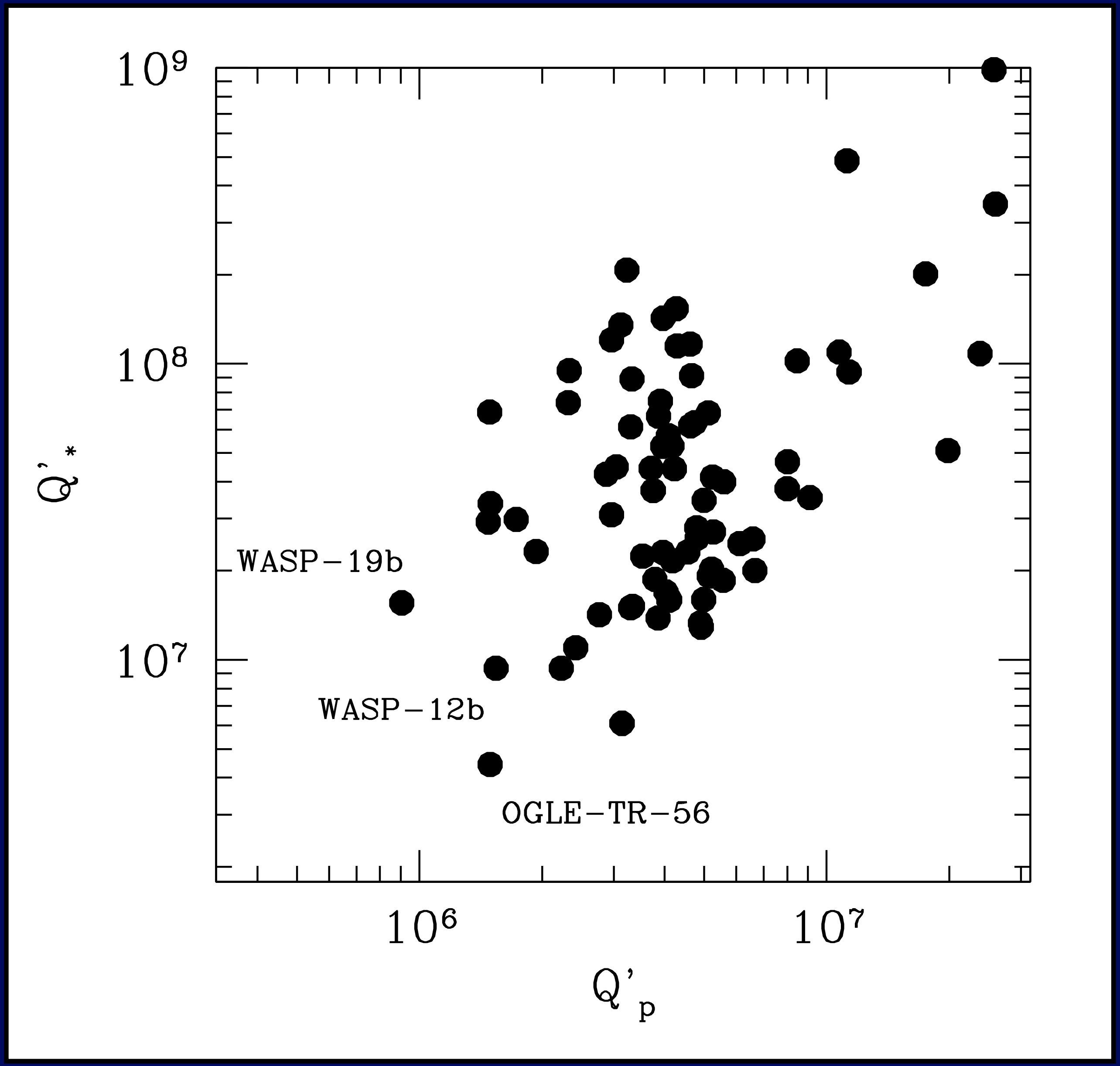
There have been claims in recent years going back and forth about the efficacy of tides in explaining the inflated radii of some hot Jupiters. Recent papers include

Jackson et al (2008) ApJ 681, 1631
Miller et al (2009) ApJ 702, 1413
Ibgui & Burrows (2009) ApJ 700,1921

There is no consensus on whether tidal heating can explain all the radii or not. With the numbers derived here, it seems difficult. Essentially, the tidal inflation occurs too early and the effects decay away on a Kelvin-Helmholtz time of 10 Myr or so....



The Q values are now dependant on system parameters



Summary

It is possible to parameterise an equilibrium tide model that describes the P-e-Mp distribution for exoplanets. The stellar tide in this model is weak enough that the model can reasonably incorporate close-in systems like WASP-19b and WASP-12b.

The price one pays for this consistency is that one cannot explain the circularisation of stellar binaries with the same stellar tidal dissipation (it is too weak). Possible explanations include restricting dissipation to convective regions (pre-MS stage is important for stellar binaries), or nonlinear effects resulting from stronger perturbations in stellar binaries (e.g. Barker & Ogilvie arXiV:1001.4009)

It is difficult to explain tidal inflation of hot Jupiter radii with this parameterisation.